Math 60

Chapter 9: Radicals and Rational Exponents

9.1 Square roots

9.2 nth root — skip rational exponents temporarily

temporarily skip 9.3

9.4 simplifying radical expressions using properties of radicals
— temporarily skip unlike indices

9.5 Adding, subtracting, multiplying, and dividing radical expressions.

9.6 Rationalizing radical expressions

9.3 { 9.2 rational exponents
9.3 simplifying using the laws of exponents }

9.3 { 9.3 simplifying using the laws of exponents
9.4 multiplying expressions with unlike indices }

9.7 Functions involving radicals

9.8 Radical equations

9.9 Complex numbers.

Math 60 9.1 Square Roots

Objectives
1) Evaluate square roots of perfect squares.
2) Approximate a square root of a number which is not a perfect square.
3) Understand that the radical symbol $\sqrt{\ }$ is called the principal square root and is always non-negative (positive or zero).
4) Determine if a square root is rational, irrational, or not a real number.
5) Find the square root of variable expressions.

The square roots of a number.
are the numbers which squared, give the original number.

① Find the square roots of 25.

$\boxed{5 \text{ and } -5}$ because $5^2 = 25$ and $(-5)^2 = 25$.

IMPORTANT
The question "find the square roots of 25" is not the same as "find $\sqrt{25}$".

② Evaluate $\sqrt{25}$.

When we use the radical $\sqrt{\ }$ symbol, we mean only the positive square root (or zero) called the principal square root.

$\sqrt{25} = \sqrt{5^2} = \boxed{5}$ only

③ Evaluate $\sqrt{0}$

$= \boxed{0}$ because $\sqrt{0^2} = 0$.

$\sqrt{\ }$ is called a radical The number inside is the radicand

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When evaluating more complicated expressions

- The radical is a grouping symbol — ("P" in PEMDAS) work from the inside out.

Evaluate.

(4) $-2\sqrt{25}$

<u>step 1:</u> grouping symbol means $\sqrt{25} = \sqrt{5^2} = 5$

$-2\sqrt{25} = -2(5)$

<u>Step 2:</u> multiply

$(-2)(5) = \boxed{-10}$

(5) $\sqrt{144} + \sqrt{25}$

$= \sqrt{12^2} + \sqrt{5^2} = 12+5$ each grouping symbol separately.

$= \boxed{17}$

(6) $\sqrt{144+25}$ inside out

$= \sqrt{169} = \sqrt{13^3}$

$= \boxed{13}$

(7) $\sqrt{49-4\cdot3\cdot2}$

grouping symbol inside first

subtract or multiply ⇒ order of operations says multiply.

$= \sqrt{49-12\cdot2}$ mult

$= \sqrt{49-24}$ subtract

$= \sqrt{25} = \sqrt{5^2}$

$= \boxed{5}$

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TRUE OR FALSE?

(8) $2\sqrt{2} = \sqrt{4}$

$\boxed{\text{FALSE}}$

$2 \cdot \sqrt{2}$ vs. $\sqrt{4} = \sqrt{2^2} = 2$

(9) $\sqrt{4} = 4$

$\boxed{\text{FALSE}}$

$\sqrt{4} = \sqrt{2^2} = 2$ vs 4

(10) $2 + 3\sqrt{7} = 5\sqrt{7}$

$\boxed{\text{FALSE}}$

order of operations
1. grouping symbols $\sqrt{7}$ first
(2. exponents)
3. divide and multiply $3 \cdot \sqrt{\ }$ second
4 subtract and add $2 +$ result last

(11) Evaluate $(\sqrt{9})^2$

order of operations: work grouping symbols from inside out

$\sqrt{9} = \sqrt{3^2} = 3$ because $3^2 = 9$

so $(\sqrt{9})^2 = 3^2 = \boxed{9}$.

(12) Evaluate $\sqrt{9^2}$

$9^2 = 81$ so $\sqrt{9^2} = \sqrt{81} = \sqrt{9^2} = \boxed{9}$

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(13) Evaluate $\sqrt{(-9)^2}$

$(-9)^2 = 81$ so $\sqrt{(-9)^2} = \sqrt{81} = \sqrt{9^2} = \boxed{9}$

(14) Evaluate $\sqrt{(8.2)^2}$

Note that square and square root "un-do" each other (mostly)

$\sqrt{(8.2)^2} = \boxed{8.2}$

(15) Evaluate

$\sqrt{\frac{1}{81}} = \sqrt{\frac{1^2}{9^2}}$ $\left(\frac{1}{9}\right)^2 = \frac{1^2}{9^2} = \frac{1}{81}$

$= \boxed{\frac{1}{9}}$

Hint: If positive numbers are in the numerator and denominator, we can split the radical into two radicals: $\sqrt{\frac{1}{81}} = \frac{\sqrt{1}}{\sqrt{81}} = \frac{1}{9}$.

(16) Evaluate $\sqrt{0.09}$

Notice $0.09 = \frac{9}{100}$

$\sqrt{0.09} = \sqrt{\frac{9}{100}} = \frac{\sqrt{9}}{\sqrt{100}} = \boxed{\frac{3}{10}} = \boxed{0.3}$

(17) Evaluate $(\sqrt{1.8})^2$

Square and square root! $= \boxed{1.8}$

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Evaluate

(18) $\sqrt{x^2}$, $x=5$

$\sqrt{5^2} = \sqrt{25} = \boxed{5}$

↑ +5 in ↑ +5 out

(19) $|x|$, $x=5$

$|5| = \boxed{5}$

Both answers are +5

Notice that $x=5$ is unchanged.

$\sqrt{5^2} = |5| = 5$

(20) $\sqrt{x^2}$, $x=-5$

$\sqrt{(-5)^2} = \sqrt{25} = \boxed{5}$

↑ −5 in ↑ +5 out

(21) $|x|$, $x=-5$

$|-5| = \boxed{5}$

Both answers are +5.

Notice that value changes

$x=-5$ goes in,

but $+5 \neq x$ comes out.

This means that for any real number a,

$$\sqrt{a^2} = |a|.$$

Evaluate. Assume variables can be any real number.

↑ This instruction means that x could be negative and absolute values are required.

(22) $\sqrt{x^2} = \boxed{|x|}$

(23) $\sqrt{(4x)^2} = \boxed{|4x|}$

(24) $\sqrt{(x-2)^2} = \boxed{|x-2|}$

(25) $\sqrt{x^2+6x+9} = \sqrt{(x+3)^2} = \boxed{|x+3|}$

(26) $\sqrt{x^2+5x+6} = \sqrt{(x+3)(x+2)}$ cannot be evaluated.

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Determine if each radical is rational, irrational, or not a real number. Evaluate each real square root, rounding to two decimal places if irrational.

$\sqrt{\text{positive perfect square}}$ — rational # $\Rightarrow$ can be written as a fraction of two integers, decimal terminates or repeats
ex. 3, $\frac{2}{3}$, $\frac{-14}{9}$

$\sqrt{\text{positive \# that's not a perfect square}}$ — irrational # $\Rightarrow$ cannot be written as a fraction of two integers, decimal does not terminate or repeat
ex. $\sqrt{2}$, $\sqrt{3}$, π

Rational $\cup$ Irrational = Real

$\sqrt{\text{negative}}$ — not a real # $\Rightarrow$ the square root of a negative # is called an imaginary #.
ex. $\sqrt{-1}$, $\sqrt{-2}$, $\sqrt{-4}$, $\sqrt{-9}$.

Not real

27. $\sqrt{31}$ $\boxed{\text{irrational}}$ $5.567 \rightarrow \boxed{5.57}$

28. $\sqrt{225}$ $\boxed{\text{rational}}$ $\boxed{15}$

29. $\sqrt{-41}$ $\boxed{\text{not a real \#}}$

With square roots, we cannot get a negative result.

$\sqrt{\text{positive}} = \text{positive}$

$\sqrt{\text{zero}} = \text{zero}$

$\sqrt{\text{negative}} = \text{not real (imaginary)}$ - section 9.9

Negative numbers are real numbers.